A Modified Synchronous Current Regulator for Brushless Motor Control

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Overview

• This work details a torque controller for brushless Permanent Magnet Synchronous Motors (PMSM).

• Methods of controlling PMSM:
  • Brushless DC Control
  • Field-Oriented Control (FOC): Synchronous Current Regulator (SCR)

• The author’s contribution is a modified SCR that:
  • uses Hall effect sensors (instead of an encoder).
  • is more computational efficient (low-cost processing).
  • has the potential for improved transient response.

• The design of the controller and an experimental application to low-cost personal transportation will be detailed.
Outline

Theoretical Analysis
• Permanent Magnet Synchronous Motor Model
• Field Oriented Control Principles
• Synchronous Current Regulator (SCR)
• Modified Synchronous Current Regulator (mSCR)

Applied Analysis
• Plant Information
• Controller Hardware
• Controller Design
• Controller Simulations: SCR and mSCR
• Experimental Testing and Data

• Future Work
• Questions / Feedback

• Motor Control Overview
• Current Sensing
• Simplified Plant Closed-Loop Transfer Function and Root-Locus
• A more fair transient response comparison.
• High-Speed Operation
• Error Handling and Failsafes
• Connection to Adaptive Feed-Forward Cancellation (AFC)
PMSM Model

Three-phase permanent magnet synchronous motor (PMSM) electromechanical model:

\[
\tau = \frac{I_a \cdot E_a + I_b \cdot E_b + I_c \cdot E_c}{\Omega}
\]

Power Conversion: $\tau = \frac{I_a \cdot E_a + I_b \cdot E_b + I_c \cdot E_c}{\Omega}$
PMSM Model

• To control torque, both the phase and the magnitude of current must be controlled.

• One option: high-bandwidth current controllers on each phase of the brushless motor. The closed-loop bandwidth must be significantly faster than the commutation of the motor (the AC frequency):

AC References:

\[ I_{xr} = |I| \sin(\omega t + \phi_x), \]
\[ x = \{a, b, c\} \]
Field-Oriented Control Principles

By exploiting symmetry of the three-phase variables and transforming to the reference frame of the rotor, the controller can act on quantities which are DC in steady-state operation.

(Similar to adaptive feed-forward cancellation with sinusoidal input.)

Field-Oriented Current control works without the need for high-bandwidth control loops.

• Easier to implement on fixed-point, low-cost microcontrollers.
• Better high-speed performance.
Field-Oriented Control Principles
Vector Motor Quantities, D/Q Axes

- Controller operates in a two-dimensional coordinate system that is attached to the rotor: rotor/synchronous reference frame.

- Direct (D) Axis: Aligned with a North magnet pole.
- Quadrature (Q) Axis: Exactly between two magnet poles.
- In a two-pole motor, they are physically perpendicular.

South-Face Magnet
North-Face Magnet
Steel
Copper Winding
Field-Oriented Control Principles
Vector Motor Quantities, D/Q Axes

- Controller operates in a two-dimensional coordinate system that is attached to the rotor: rotor reference frame.

- Direct (D) Axis: Aligned with a North magnet pole.

- Quadrature (Q) Axis: Exactly between two magnet poles.

- The axes are attached to the rotor. Q always leads D in the direction of rotation.
Field-Oriented Control Principles
Vector Motor Quantities, D/Q Axes

- Controller operates in a two-dimensional coordinate system that is attached to the rotor: rotor reference frame.

- Direct (D) Axis: Aligned with a North magnet pole.

- Quadrature (Q) Axis: Exactly between two magnet poles.

- In a four-pole motor, they are separated by 45° mechanical. They are always separated by 90° electrical.
Field-Oriented Control Principles
Vector Motor Quantities, D/Q Axes

- All motor quantities that have “direction” can be projected onto the d/q axes as vectors:

  Stator Current / Flux: Vector sum of coil current/flux defined by right hand rule.

  Back EMF: Always on the q-axis.
  \[ E = \frac{d\lambda}{dt} \]

  Rotor Flux Linkage: Always on the d-axis for a permanent magnet motor.
  \[ \lambda = N\phi \]

\[ \tau = \overrightarrow{I} \cdot \overrightarrow{E} \]

South-Face Magnet
North-Face Magnet
Steel
Copper Winding
Field-Oriented Control Principles
Unrealistic Zero-Inductance Motor

- Voltage applied in-phase with back-EMF.
- Current also in-phase with back-EMF.
- Torque per amp is optimal.
- Reasonable approximation if inductance or speed is low:
  \[ \omega L \ll R \]
Field-Oriented Control Principles
Motor with Inductance

- Voltage applied in-phase with back-EMF.
- Current lags due to the motor inductance.
- Torque per amp is no longer optimal. Current and back EMF are not in phase:

$$\vec{I} \times \vec{E} \neq 0$$
Field-Oriented Control Principles
Phase Advance to Correct for Inductance Lag

- Voltage applied ahead of back EMF.
- Current lags due to the motor inductance such that it is in phase with back EMF.
- Torque per amp is optimal.

\[ \phi = f(V, I, \Omega, K_t, R, L, \ldots) \]
Field-Oriented Control Principles
Field Weakening for High-Speed Operation

- Voltage and current both lead back EMF.
- Stator flux counteracts rotor flux: “field weakening”
- Torque per amp is not optimal but…
- Maximum achievable speed per volt is higher.
Field-Oriented Control Principles
Park Transform / Inverse Park Transform

- Transforms used to convert from/to stator frame \{a,b,c\} quantities to/from rotor frame \{d,q\} quantities.
- Require rotor position, \( \theta \), as an input.

\[
\begin{align*}
\begin{bmatrix} x_d \\ x_q \\ x_0 \end{bmatrix} &= T \begin{bmatrix} x_a \\ x_b \\ x_c \end{bmatrix} \\
T &= \frac{2}{3} \begin{bmatrix}
\cos \theta & \cos(\theta - \frac{2\pi}{3}) & \cos(\theta + \frac{2\pi}{3}) \\
-\sin \theta & -\sin(\theta - \frac{2\pi}{3}) & -\sin(\theta + \frac{2\pi}{3}) \\
\frac{1}{2} & \frac{1}{2} & \frac{1}{2}
\end{bmatrix}
\end{align*}
\]

\[
\begin{align*}
\begin{bmatrix} x_a \\ x_b \\ x_c \end{bmatrix} &= T^{-1} \begin{bmatrix} x_d \\ x_q \\ x_0 \end{bmatrix} \\
T^{-1} &= \begin{bmatrix}
\cos \theta & -\sin \theta & 1 \\
\cos(\theta - \frac{2\pi}{3}) & -\sin(\theta - \frac{2\pi}{3}) & 1 \\
\cos(\theta + \frac{2\pi}{3}) & -\sin(\theta + \frac{2\pi}{3}) & 1
\end{bmatrix}
\end{align*}
\]
Synchronous Current Regulator

- Park and inverse Park transform convert into and out of rotor reference frame.
- Two “independent” controllers for the d- and q-axis.
- Requires rotor position, typically from an encoder or resolver.
Synchronous Current Regulator

- Because the controllers run in the rotor frame, where values are “DC” in steady state, the controllers may operate at low bandwidth, below commutation frequency, and long time-constant current filtering can be implemented.
Modified Synchronous Current Regulator

Initial Motivation

- For sufficient resolution of rotor position, an encoder or resolver is typically required for field oriented control. (Sensorless techniques also exist.)
- However, less expensive motors use three Hall effect sensors to derive rotor position with 60° electrical resolution:
Modified Synchronous Current Regulator

Initial Motivation

In sensored brushless DC control, the six Hall effect sensor states directly map to phase voltage outputs.

### State Table

<table>
<thead>
<tr>
<th>State</th>
<th>$V_a$</th>
<th>$V_b$</th>
<th>$V_c$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>PWM</td>
<td>0V</td>
<td>High-Z</td>
</tr>
<tr>
<td>2</td>
<td>High-Z</td>
<td>0V</td>
<td>PWM</td>
</tr>
<tr>
<td>3</td>
<td>0V</td>
<td>High-Z</td>
<td>PWM</td>
</tr>
<tr>
<td>4</td>
<td>0V</td>
<td>PWM</td>
<td>High-Z</td>
</tr>
<tr>
<td>5</td>
<td>High-Z</td>
<td>PWM</td>
<td>0V</td>
</tr>
<tr>
<td>6</td>
<td>PWM</td>
<td>High-Z</td>
<td>0V</td>
</tr>
</tbody>
</table>

- **Pros:** very simple algorithm (state table), can run on low-cost processor.
- **Cons:** fixed timing, torque ripple, audible noise

Initial Motivation: Can the Synchronous Current Regulator be modified to work with Hall effect sensor inputs, with interpolation?
Modified Synchronous Current Regulator

Slow Loop (100-1,000Hz)

- 0 or $I_{dr}$
- $I_{qr}$

Fast Loop (10kHz)

- Hall Effect Interpolator
- Hall Effect Sensors
- Synchronous Measurement

- $d$-axis controller
- $q$-axis controller
- Park Transform

- $I_d = \frac{1}{\tau s + 1}$
- $I_q = \frac{1}{\tau s + 1}$
- $I_c = -I_a - I_b$

- PWM
- $\theta$
- $\phi$
- $|V|$
Modified Synchronous Current Regulator

There are several practical differences:

- The controller is explicitly split into fast and slow loops; only PWM generation and rotor position estimation need be in the fast loop.
- PWM generation is done by a sine table look-up, which is faster to compute than an inverse Park transform.
- The rotor position is estimated by interpolating between Hall effect sensor absolute states using the last known speed.
- As long as rotor position and phase currents are sampled synchronously by the slow loop, the slow loop bandwidth can be arbitrarily low.

- The modified synchronous current regulator can be run on fixed-point processors to control low-cost motors with Hall effect sensors.
- It can achieve AC servo motor-like control with brushless DC motors.
Modified Synchronous Current Regulator

The primary *theoretical* difference is the controller outputs:

**Standard SCR**
- $V_d$ and $V_q$ fully-define a voltage vector.
- D-axis gain: $[V/A]$
- Q-axis gain: $[V/A]$

  - Simulate with:

**Modified SCR**
- $|V|$ and $\angle V$ fully-define a voltage vector.
- D-axis gain: $[\text{rad}/A]$
- Q-axis gain: $[V/A]$

  - Simulate with:
Modified Synchronous Current Regulator

Consider a step increase in torque command via $I_{qr}$:

$$Q, V_d, V_q, \Delta V_{mSCR}, \Delta V_{SCR}, I\omega L, IR, E, I, \phi$$

SCR:

$$0 + \frac{K_d}{s} V_d$$

$$I_q, I_d$$

mSCR:

$$0 - \frac{K_d}{s} \phi$$

$$|V|$$

$$I_q, I_d$$
Applied Analysis
Plant Information
Overview

• The controller presented here has been tested on several plants.
• The example used for this presentation is a 500W electric kick scooter.

• Custom-designed and built hub motor.
• Rear wheel direct drive, 1:1.
• 33V, 4.4Ah LiFePO4 battery.
• Torque command by hand throttle.
## Plant Information
### Important Specifications

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
<th>Value</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>$2p$</td>
<td>Number of poles.</td>
<td>14</td>
<td>-</td>
</tr>
<tr>
<td>$R_a$</td>
<td>Per-phase motor resistance.</td>
<td>0.084</td>
<td>Ω</td>
</tr>
<tr>
<td>$L_s$</td>
<td>Synchronous inductance.</td>
<td>$0.2 \times 10^{-3}$</td>
<td>H</td>
</tr>
<tr>
<td>$K_t$</td>
<td>Per-phase torque/back EMF constant.</td>
<td>0.10</td>
<td>V/(rad/s)</td>
</tr>
<tr>
<td>$V$</td>
<td>Nominal DC voltage.</td>
<td>33.0</td>
<td>V</td>
</tr>
<tr>
<td>$J$</td>
<td>Plant inertia, reflected to rotational.</td>
<td>0.40</td>
<td>kg⋅m²</td>
</tr>
</tbody>
</table>
Controller Hardware
Overview

• Custom 48V/40A three-phase inverter drive
• Hall effect-based current sensing (phase and DC).
• v1,2: Texas Instruments MSP430F2274 (16-bit, no hardware multiplier)
  v3: STMicroelectronics STM32F103 (32-bit, w/ hardware multiplier)
• 2.4GHz wireless link for data acquisition.
## Controller Hardware
### Important Specifications

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
<th>Value</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>$R_{ds}$</td>
<td>On-resistance of each phase leg.</td>
<td>$7.5 \times 10^{-3}$</td>
<td>Ω</td>
</tr>
<tr>
<td>$f_{sw}$</td>
<td>PWM switching frequency.</td>
<td>15,625</td>
<td>Hz</td>
</tr>
<tr>
<td>$f_{fast}$</td>
<td>Fast-loop frequency. Handles position estimate, sine wave generation.</td>
<td>MSP430: 14,500 STM32: 10,000</td>
<td>Hz</td>
</tr>
<tr>
<td>$f_{slow}$</td>
<td>Slow-loop frequency. Handles current sampling, control computation.</td>
<td>MSP430: 122 STM32: 1,000</td>
<td>Hz</td>
</tr>
<tr>
<td>$f_{tx}$</td>
<td>Data transmit frequency. For data display and logging.</td>
<td>20</td>
<td>Hz</td>
</tr>
</tbody>
</table>
Controller Design
Overview

Synchronous Current Regulator:

\((I_{dr} - I_d)\)

D-Axis Controller

\((I_{qr} - I_q)\)

Q-Axis Controller

\(V_d\)

\(dq\)

\(V_q\)

abc

\(V_{\{abc\}}\)

Controllers  Inverse Park Transform  Amplifier  Motor

Modified Synchronous Current Regulator:

\((I_{dr} - I_d)\)

D-Axis Controller

\((I_{qr} - I_q)\)

Q-Axis Controller

\(\angle V\)

|V|

\(V_{\{abc\}}\)

Controllers  Sine Wave Generator  Amplifier  Motor
Controller Design
Simplified Plant: Q-Axis Only, Stalled

- At stall, both the d-axis and the q-axis look like resistors.
- Modeling the q-axis (torque-producing) controller and plant:

\[
\begin{align*}
I_{qr} & \rightarrow \frac{K_q}{s} I_{qe} \rightarrow V_q \rightarrow \frac{1}{R_a} \rightarrow I_q \\
G_c(s) & \quad G_p(s) \\
H(s) & = \frac{1}{\tau_d s + 1}
\end{align*}
\]

- Closed-loop poles can be placed anywhere in the left half-plane, bandwidth set by filter frequency and damping ratio set by \(K_q\).
Controller Design
Simplified Plant: Q-Axis Only, Stalled

\[ L(s) = \frac{1}{(R_a s)(\tau_d s + 1)} \]

To leave 75° phase margin:

\[ K_q \approx 1.2 \frac{V}{A \cdot s} \]

\[ \phi_m = 75^\circ \]

\[ \zeta \approx 0.75 \]
Controller Design
Simplified Plant: Q-Axis Only, Stalled

Simplified Plant Closed-Loop Step Response

- $K_q = 1.2 \text{ V/A/s}$
- $K_q = 1.6 \text{ V/A/s}$
- $K_q = 2.5 \text{ V/A/s}$

Amplitude

Normalized $I_q$
Controller Simulations
Synchronous Current Regulator

- Full motor simulation with vector quantities and complex impedance using measured motor parameters ($R_a$, $L_s$, $K_t$).
- Current filtering as described above.
- Speed fixed at 500rpm. (Load dynamics not considered.)
- $I_{dr} = 0$, $I_{qr}$ steps from 15A to 30A.

\[ V_d = V_q + K_d \frac{V_d}{s}, \quad K_d = \{1.2, 1.6, 2.5\} \text{ V/A/s} \]

\[ V_q = V_d - K_q \frac{V_q}{s}, \quad K_q = \{1.2, 1.6, 2.5\} \text{ V/A/s} \]
Controller Simulations
Synchronous Current Regulator

SCR: Vector Step Response, Voltage

- $K_q = K_d = 1.2 \text{ V/A/s}$
- $K_q = K_d = 1.6 \text{ V/A/s}$
- $K_q = K_d = 2.5 \text{ V/A/s}$
Controller Simulations
Synchronous Current Regulator

What am I looking at?
Controller Simulations
Synchronous Current Regulator

SCR: Vector Step Response, Voltage

- $K_q = K_d = 1.2$ V/A/s
- $K_q = K_d = 1.6$ V/A/s
- $K_q = K_d = 2.5$ V/A/s
Controller Simulations
Synchronous Current Regulator

SCR: Vector Step Response, Current

$K_q = K_d = 1.2$ V/A/s

$K_q = K_d = 1.6$ V/A/s

$K_q = K_d = 2.5$ V/A/s
Controller Simulations
Synchronous Current Regulator

SCR: Step Response, Torque

- $K_q = K_d = 1.2 \text{ V/A/s}$
- $K_q = K_d = 1.6 \text{ V/A/s}$
- $K_q = K_d = 2.5 \text{ V/A/s}$
Controller Simulations
Modified Synchronous Current Regulator

- Full motor simulation with vector quantities and complex impedance using measured motor parameters ($R_a$, $L_s$, $K_t$).
- Current filtering as described above.
- Speed fixed at 500rpm. (Load dynamics not considered.)
- $I_{d} = 0$, $I_{q}$ steps from 15A to 30A.

$K_d = 1.0 \text{ rad/A/s}$  
$K_q = \{1.2, 1.6, 2.5\} \text{ V/A/s}$
Controller Simulations
Modified Synchronous Current Regulator

mSCR: Vector Step Response, Voltage

- $K_q = 1.2 \text{ V/A/s}$
- $K_q = 1.6 \text{ V/A/s}$
- $K_q = 2.5 \text{ V/A/s}$
Controller Simulations
Synchronous Current Regulator

What am I looking at?
Controller Simulations
Modified Synchronous Current Regulator

mSCR: Vector Step Response, Voltage

- $V_d$ [V]
- $V_q$ [V]

- $K_q = 1.2$ V/A/s
- $K_q = 1.6$ V/A/s
- $K_q = 2.5$ V/A/s
Controller Simulations
Modified Synchronous Current Regulator

mSCR: Vector Step Response, Current

- $I_d$ [A]
- $I_q$ [A]

- $K_q = 1.2$ V/A/s
- $K_q = 1.6$ V/A/s
- $K_q = 2.5$ V/A/s
Controller Simulations
Modified Synchronous Current Regulator

mSCR: Step Response, Torque

\[ K_q = 1.2 \text{ V/A/s} \]
\[ K_q = 1.6 \text{ V/A/s} \]
\[ K_q = 2.5 \text{ V/A/s} \]
Controller Simulations
Comparison

**Voltage**

SCR: Vector Step Response, Voltage

![Voltage Graph](image)

**Current**

SCR: Vector Step Response, Current

![Current Graph](image)

**Torque**

SCR: Step Response, Torque

![Torque Graph](image)
Experimental Testing and Data
Baseline: Q-axis Control Only

- Q-axis (torque producing) current controlled.
- D-axis current increases with speed.
Experimental Testing and Data
Baseline: Q-axis Control Only

- Q-axis (torque producing) current controlled.
- D-axis current increases with speed.
Experimental Testing and Data

Full mSCR

- D-axis current controlled to be zero.
- Phase advanced as speed increases.
Experimental Testing and Data

Full mSCR

- In the positive torque quadrant, $I_d$ is effectively regulated.
- Negative torque still needs work, but it’s better than open-loop.
Future Work

• Range testing (or directly measure energy consumption) with SCR vs. mSCR in real-world use.

• Controlled dynamometer experiment of SCR vs. mSCR transient torque response, to verify simulations. (Requires high-speed data acquisition.)

• Sensorless control using a state observer for rotor position.

• Fault detection and recovery to increase controller robustness, possibly using sensorless control as a “back-up” in the event of sensor failure.

• More high-speed testing.

• Larger-scaled motor and controllers.
Questions / Feedback
References


Motor Control Overview

- Electric motors convert electrical power (voltage, current) to mechanical power (torque, speed), with some power lost as heat in the motor.

![Brushed DC Motor Model]

- The torque constant \((K_t)\) and back EMF constant are identical due to power conservation. The conversion from current and back EMF to torque and speed is lossless; all losses are accounted for externally.
Motor Control Overview

- A *brushed* DC motor can be modeled as a SISO system (voltage to speed) with an internal feedback loop of back EMF:
Motor Control Overview

• A current control loop provides the ability to command torque. Current is directly proportional to torque, and easy to measure.

• Depending on the load, an integral controller may be sufficient to track the reference current with zero steady-state error.
Current Sensing
Overview

Digital LPF  Park Transform  $I_a + I_b + I_c = 0$  Analog LPF

<table>
<thead>
<tr>
<th>Digital</th>
<th>Analog</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rotor Position Estimator</td>
<td></td>
</tr>
<tr>
<td>θ</td>
<td>Trigger</td>
</tr>
<tr>
<td>Latch Value</td>
<td></td>
</tr>
<tr>
<td>θ_{latch}</td>
<td></td>
</tr>
<tr>
<td>$I_d$</td>
<td>$I_a$</td>
</tr>
<tr>
<td>$I_q$</td>
<td>$I_c$</td>
</tr>
<tr>
<td>$abc$</td>
<td>dq</td>
</tr>
</tbody>
</table>

1kHz Sampling
**Current Sensing**  
**Analog Filtering: Second-Order Low Pass**

1. Buffered output filter on ACS714 Hall effect current sensor.  
2. Local 2:1 voltage divider and RC filter at ADC pin.

\[ F(s) = \frac{1}{\tau_1 s + 1} \frac{1}{\tau_2 s + 1} \]

\[
\tau_1 = (1.7k\Omega)(C_F)\\
\tau_2 = \frac{1}{2} R_2 C_2
\]
Current Sensing
Analog Filtering: Second-Order Low Pass

- The goal is to do as little filtering of the AC current signal as possible, so as not to distort the phase of the current. (Less than 5° phase lag desirable.)
- The PWM frequency (15,625Hz) is an obvious target for filtering.
  1. Actual current ripple will be at this frequency.
  2. Power transient-induced noise will be here, too.
- The filtering after the Park Transform can be much more aggressive, so noise in the AC current signal is acceptable.
- Component Selection:

\[
F(s) = \left( \frac{1}{\tau_1 s + 1} \right) \left( \frac{1}{\tau_2 s + 1} \right)
\]

\[
C_F = 10nF
\]
\[
R_2 = 10k\Omega
\]
\[
C_2 = 10nF
\]
\[
\tau_1 = (1.7k\Omega) (10nF) = 17\mu s
\]
\[
\tau_2 = \frac{1}{2} (10k\Omega)(10nF) = 50\mu s
\]
Current Sensing
Analog Filtering: Second-Order Low Pass

Current Sensor AC Filter, $F(s)$

- **Magnitude (dB)**
  - -100
  - -80
  - -60
  - -40
  - -20
  - 0
  - 20
  - 40
  - 60
  - 80
  - 100

- **Phase (deg)**
  - -180
  - -135
  - -90
  - -45
  - 0
  - 45
  - 90
  - 135
  - 180

- **Frequency (rad/sec)**
  - $10^2$
  - $10^3$
  - $10^4$
  - $10^5$
  - $10^6$
  - $10^7$

- **Notes**
  - PWM Frequency
    - -20dB Filtering
  - Maximum Commutation Frequency
    - 4º Phase Lag
Current Sensing
Digital Filtering: First-Order Low Pass

• The digital filter acts on \( I_d \) and \( I_q \), the outputs of the Park transform.
• At steady-state, these are DC quantities. The *filter time constant can be much slower than the commutation frequency*.
• The bandwidth lower limit is driven by the target performance of the current (torque) controller.
• The bandwidth upper limit is driven by the sampling frequency. The filter time constant should be much longer than the sampling interval.
• Where \( \Delta t \) is the sampling interval, a first-order digital low pass filter on \( I_d \) and \( I_q \) can be implemented with the following difference equations:

\[
I_q^n = a \cdot I_q^{n-1} + (1 - a) \cdot I_q'
\]
\[
I_d^n = a \cdot I_d^{n-1} + (1 - a) \cdot I_d'
\]

Equivalent continuous time constant:

\[
\tau_d = \frac{a}{1 - a} \Delta t
\]
Current Sensing
Digital Filtering: First-Order Low Pass

- Parameter Selection:

\[ \Delta t = 1\, ms \]
\[ a = 0.95 \]
\[ \tau_d = \left( \frac{a}{1 - a} \right) \Delta t = \left( \frac{0.95}{0.05} \right) 1\, ms = 19\, ms \]

- The filter time constant is significantly longer than the sampling interval, so a “continuous time” analysis is appropriate:

\[ H(s) \approx \frac{1}{\tau_d s + 1} \]

- The bandwidth is \( 1/\tau_d \), 52.6rad/s, or 8.38Hz.
Simplified Plant
Closed-Loop Transfer Function and Root Locus

\[ L(s) = G_c G_p H = \frac{K_q}{R_a s (\tau_d s + 1)} \]

\[ G_{cl}(s) = \frac{G_c G_p}{1 + G_c G_p H} = \frac{K_q \tau_d s + K_q}{R_a \tau_d s^2 + R_a s + K_q} \]

\[ \zeta = 0.707 \]
Controller Simulations
A more fair transient response comparison.

One possible way to make a more fair comparison is by using the initial voltage vector to normalize the new d-axis gain:

\[ K_d = \frac{1.0 \text{ rad/A/s}}{|V_0|} \]
Controller Simulations
A more fair transient response comparison.

SCR: Vector Step Response, Voltage

- $K_q = K_d = 1.2 \text{ V/A/s}$
- $K_q = K_d = 1.6 \text{ V/A/s}$
- $K_q = K_d = 2.5 \text{ V/A/s}$

mSCR Vector Step Response, Voltage

- $K_q = 1.2 \text{ V/A/s}$
- $K_q = 1.6 \text{ V/A/s}$
- $K_q = 2.5 \text{ V/A/s}$
Controller Simulations
A more fair transient response comparison.

SCR: Vector Step Response, Current

- $K_q = K_d = 1.2 \text{ V/A/s}$
- $K_q = K_d = 1.6 \text{ V/A/s}$
- $K_q = K_d = 2.5 \text{ V/A/s}$

mSCR Vector Step Response, Current

- $K_q = 1.2 \text{ V/A/s}$
- $K_q = 1.6 \text{ V/A/s}$
- $K_q = 2.5 \text{ V/A/s}$
Controller Simulations
A more fair transient response comparison.

![SCR Step Response, Torque](image1)

![mSCR Step Response, Torque](image2)
High Speed Operation

• Sensing and control becomes more difficult as speed increases:
  • $\omega L \approx R$, large phase angle.
  • Significant lag due to current sensing / AC-side filtering.
  • Analysis of digital effects (sampling, filtering) becomes important.

• Poles: 2

• Max Speed: 35,000RPM (without field weakening)
  $\omega = 3,665 \text{rad/s}, f = 583 \text{Hz}$

• Current sensor phase lag with components specified: $\sim 20^\circ$!
High Speed Operation
Error Handling and Failsafes

• Hall effect sensor failure presents a significant risk to the controller.

<table>
<thead>
<tr>
<th>Failure Mode</th>
<th>Effect</th>
<th>Countermeasure</th>
</tr>
</thead>
<tbody>
<tr>
<td>The entire sensor cable becomes unplugged.</td>
<td>Complete loss of ability to commutate the motor.</td>
<td>Pull-down resistors take the sensor state to {0,0,0}, which is invalid. The output driver shuts down. Motor coasts.</td>
</tr>
<tr>
<td>Transient sensor glitch. &lt; 1/6 cycle (single sensor glitch)</td>
<td>An unexpected state transition, resulting in large current/torque transient when voltage vector is applied at the wrong angle.</td>
<td>If new state is not as expected, trust rotor speed interpolation for the next 60° segment.</td>
</tr>
<tr>
<td>Permanent sensor failure. &gt; 1/6 cycle</td>
<td>Repeated loss of two states per cycle.</td>
<td>Follow same rules as above, but with a counter that tallies unexpected state transitions per unit time. If larger than some threshold, shut down.</td>
</tr>
</tbody>
</table>

... 

• Sensorless or hybrid techniques will significantly change the FMEA.

• Future work: Ability to switch to sensorless control if a Hall effect sensor fault is detected.
Connection to Adaptive Feed-Forward Cancellation

- The SCR and mSCR are applications of adaptive feed-forward cancellation (AFC) to three-phase variables.
- In one implementation of AFC, a feed-forward path allows for zero-error tracking of a sinusoidal input at a specific frequency:

Reference:
Connection to Adaptive Feed-Forward Cancellation

- By manipulating the block diagram of a the SCR, focusing on the amplitude of a single phase of current, the SCR can be related to single-oscillator AFC (not proven here).

- The modified SCR is related to single-oscillator AFC with a phase advance offset, which has been proven to improve transient response.

- In both cases, the Park Transform provides the sinusoidal multiplier for the input and output.

- In AFC with phase advance, $\phi_i$ is set as the plant phase angle (initial voltage vector angle).

Reference: